

**TMATYC**  
**CALCULUS TEST**  
 Spring 2003

1. What is the value of  $A$  which will make the function  $f(x) = \begin{cases} Ax+1 & x < 2 \\ 3x^2 - 5 & x \geq 2 \end{cases}$  continuous at  $x = 2$ ?

A. -1                      B. 4                      C. 3                      D. -2

2. Find the derivative of the function:  $f(x) = \frac{-1}{(2x-1)^{99}}$

A.  $f'(x) = \frac{198}{(2x-1)^{100}}$                       B.  $f'(x) = \frac{-99}{(2x-1)^{98}}$

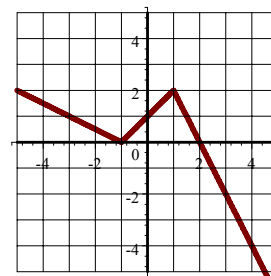
C.  $f'(x) = \frac{99}{2(2x-1)^{100}}$                       D.  $f'(x) = \frac{-198}{(2x-1)^{98}}$

3. Which of the following values qualifies as a “critical point” for the function  $f(x) = 4 \sin(2x) + 4x$  on the interval  $[0, \pi]$  ?

A.  $x = \frac{3\pi}{8}$                       B.  $x = \frac{3\pi}{4}$                       C.  $x = \frac{2\pi}{3}$                       D.  $x = \frac{\pi}{3}$

4. The graph of the function  $h$  is shown at the right. Use it to estimate the value of  $h'(2)$ .

A. -2                      B. 0  
 C. 1                      D. does not exist



5. A fish starting at rest moves across a pond with a constant acceleration of  $5 \text{ ft/sec}^2$ . How far does he travel in four seconds?

A. 40 feet                      B. 30 feet                      C. 20 feet                      D. 10 feet

6. Find the area of the triangular region formed in the first quadrant bounded above by the line tangent to the curve  $y = 2x^2 - 6x + 5$  at the point  $(1, 1)$ .

- A.  $\frac{9}{4}$                       B. 9                      C.  $\frac{18}{5}$                       D.  $\frac{4}{3}$

7. Evaluate  $\int_0^1 3x \cos(2x^2) dx$

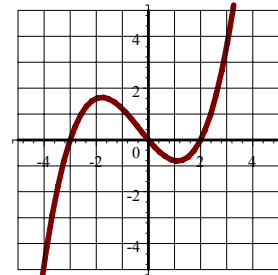
- A.  $0.75\sin(1)$                       B.  $4\sin(1)$                       C.  $0.75\sin(2)$                       D.  $\sin(2)$

8. Calculate the volume of the solid generated by revolving the graph of  $y = x^2 + 1$  about the x-axis between the values of  $x = 0$  and  $x = 2$ .

- A.  $\frac{227\pi}{5}$                       B.  $\frac{206\pi}{15}$                       C.  $\frac{251\pi}{15}$                       D.  $\frac{109\pi}{5}$

9. Let  $g(x) = \int_{-5}^x f(t) dt$  where the graph of  $f$  is shown at the right. Estimate when  $g$  has a relative maximum.

- A. -3                      B. -2  
C. 0                      D. 1



10. Find the dimensions of the largest rectangle which can be inscribed underneath the parabola  $y = 8 - x^2$  and bounded below by the x-axis.

- A.  $\frac{8}{3}$  by  $8\sqrt{3}$                       B.  $\frac{16}{3}$  by  $\frac{4\sqrt{6}}{3}$                       C. 4 by 4                      D.  $\frac{8\sqrt{2}}{3}$  by  $\frac{2}{3}$

11. Find the equation of the line tangent to the curve  $x(t) = 2t - 3$ ,  $y(t) = \frac{3}{t}$  at  $t = 3$ .

- A.  $6x + 2y = 16$                       B.  $x + 6y = 9$                       C.  $2x - 4y = 2$                       D.  $3x - 8y = 1$

12. Find the value of the limit:  $\lim_{x \rightarrow 2} \frac{||x| - 2|}{(x-2)^2}$

- A. 0                      B.  $-\infty$                       C.  $\infty$                       D. does not exist

13. Integrate  $\int \frac{\tan x}{\csc x} dx$

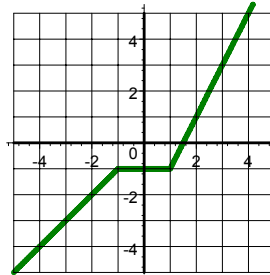
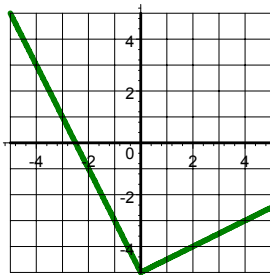
A.  $\ln|\sec x| + \cos x + C$

B.  $-\ln|\sin x| - \cos x + C$

C.  $\ln|\sec x + \tan x| - \sin x + C$

D.  $\frac{x}{2} - \frac{\sin 2x}{4} + C$

14. The graph of the function  $no(x)$  is shown below left and the graph of  $oh(x)$  is shown below right. If  $f(x) = oh(no(x))$  estimate the value of  $f'(2)$ .



A. -2

B. 1

C. 0

D. 2

15. Find  $\frac{dy}{dx}$  for the relation:  $x + y \cos(2x) = y^2$

A.  $\frac{x \cos(2x) + xy^2}{y^2 \sin(2x)}$

B.  $\frac{\cos(2x) - y^2}{\sin(2x) - xy}$

C.  $\frac{x - 2 \cos(2x)}{y + \sin(2x)}$

D.  $\frac{1 - 2y \sin(2x)}{2y - \cos(2x)}$

16. The velocity of a particle moving along the  $x$ -axis is given by the function  $v(t) = 3t^2 - 18t + 24$ , where  $t \geq 0$ . The initial position of the particle is  $(1, 0)$ . What is the total distance the particle travels in the first 4 seconds?

A. 24 units

B. 17 units

C. 21 units

D. 16 units

17. Find the area under the curve  $f(x) = 4xe^{x^2}$  between  $x = 0$  and  $x = 1$ .

A.  $4e - 2$

B.  $2e - 1$

C.  $4e$

D.  $2e - 2$

18. Evaluate the definite integral:  $\int_0^{1/2} \frac{1}{\sqrt{1-x^2}} dx$ .

A.  $\frac{\pi}{2}$

B.  $\frac{\pi}{3}$

C.  $\frac{\pi}{4}$

D.  $\frac{\pi}{6}$

19. Find the sum of the series:  $\sum_{n=0}^{\infty} \frac{3^{n+1}}{5^n}$

- A. 4                      B. 5.5                      C. 6                      D. 7.5

20. Integrate:  $\int \frac{2}{x^2 + x} dx$

- A.  $\ln\left(\frac{x}{x+1}\right)^2 + C$                       B.  $2\ln|x^2 + x| + C$   
 C.  $2\tan^{-1}\left(x + \frac{1}{2}\right) + C$                       D.  $\frac{\ln(x^2 + x)}{x^2 + x} + C$

21. Two cars leave the same location at the same time. The red car heads due east at a constant speed of 45 miles per hour. The blue car heads due south at a constant rate of 60 miles per hour. What is the rate of change of the distance between the two cars 2 hours later?

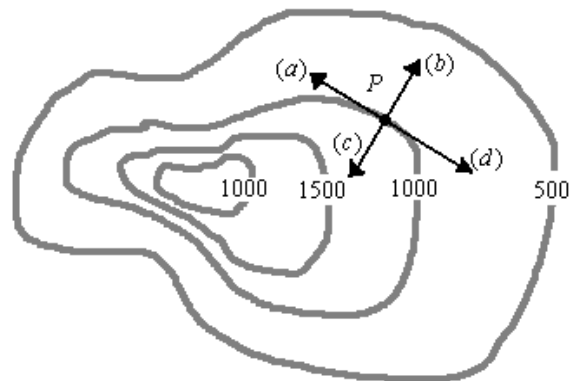
- A. 55 mph                      B. 65 mph                      C. 75 mph                      D. 85 mph

22. Evaluate the limit:  $\lim_{x \rightarrow \infty} \left(1 - \frac{2}{x}\right)^{2x}$

- A.  $e^{-2}$                       B.  $e$                       C.  $e^{-4}$                       D.  $e^2$

23. The level curves for the function  $z = f(x, y)$  are shown here. Which vector indicates the direction of  $\nabla f$  at the point  $P$ ?

- A. Vector (a)                      B. Vector (b)  
 C. Vector (c)                      D. Vector (d)



24. The plane containing the points  $(0, 0, 1)$ ,  $(0, 1, 1)$  and  $(1, 0, 1)$  is normal to which vector?

- A.  $\mathbf{k}$                       B.  $-\mathbf{i} + \mathbf{j}$                       C.  $\mathbf{i} - \mathbf{k}$                       D.  $-\mathbf{j}$

25. Solve the equation  $y' = y^2$ .

- A.  $y = Ce^{2x}$                       B.  $y = \frac{-1}{x+C}$                       C.  $y = 2\ln|x| + C$                       D.  $y = \cos\left(\frac{x}{2}\right) + C$

## Answer Key

1. C
2. A
3. D
4. A
5. A
6. A
7. C
8. B
9. C
10. B
11. B
12. C
13. C
14. B
15. D
16. A
17. D
18. D
19. D
20. A
21. C
22. C
23. C
24. A
25. B

1.